$$F(a,\ a,\ d,\ d)=f(a,\ a,\ d,\ d)f'(a,\ a,\ d,\ d)=12a^2d^2-2a^4-2d^4.$$

That is, $12a^2d^2-2a^4-2d^4$ must be resoluble into two rational factors in a and d, since neither $f(a,\ a,\ d,\ d)$ nor $f'(a,\ a,\ d,\ d)$ can equal unity. It is evident however that $12a^2d^2-2a^4-2d^4$ does not possess this property.

GEOMETRY.

190. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Find the locus of the centers of sections of an ellipsoid by planes which are at a constant distance from the center.

Solution by the PROPOSER.

The center of the ellipsoid being the origin, and $(\alpha,\ \beta,\ \gamma)$ being the center of the section, its equation is found to be

$$\frac{\alpha}{a^2}x+\frac{\beta}{b^2}y+\frac{\gamma}{c^2}z-\left(\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}+\frac{\gamma^2}{c^2}\right)=0....(1).$$

The perpendicular from the center of the ellipsoid upon it is

$$\left(\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}+\frac{\gamma^2}{c^2}\right)\div\sqrt{\left(\frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}+\frac{\gamma^2}{c^4}\right)}=k,$$

a constant, by the problem. This gives the required locus, which, by rationalizing, is easily seen to be a surface of the fourth degree.

Excellent solutions were also received from *PROFESSORS ZERR, WALKER*, and *SCHEFFER*.

CALCULUS.

151. Proposed by J. SCHEFFER. A. M., Hagerstown, Md.

Integrate the differential equation, $xy\dfrac{\partial^2 z}{\partial x.\partial y}=bx\dfrac{\partial z}{\partial x}+ay$.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let $x=e^u$, $y=e^v$; then with $x\dfrac{d}{dx}=\theta$, $y\dfrac{d}{dy}=\theta'$, the given equation reduces to $\theta(\theta'-b)z=ae^v....(1)$, in which u and v are the independent variables.

The integral of (1) is

$$z=c\phi(y)+\frac{ay}{1-b}\log x+c_1\log x+c_2\ldots.(2),$$

noticing that $u=\log x$ and $v=\log y$.

Also solved by *G. B. M. ZERR*, *G. R. DEAN*, *L. C. WALKER*, and *J. SCHEFFER*.
Professor Walker should have been credited with a solution of Problem 148.

152. Proposed by F. P. MATZ. Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Solve the differential equation, $e^x\left[\frac{dy}{dx}-y\log x\right]-a[\log x+1]=0$.

Solution by BEULAH FRAIZER, Sophomore Student, Missouri School of Mines, Rolla, Mo.

Dividing by e^x, $\frac{dy}{dx}-y\log x=ae^{-x}[\log x+1]$. The integrating factor is $e^{-\int\log x dx}=e^{+x(1-\log x)}=\frac{e^x}{x^x}$. Multiplying by this,

$$\frac{e^x}{x^x}\frac{dy}{dx}-\frac{y\log x e^x}{x^x}=a\frac{\log x+1}{x^x}y.$$

The left hand member is the derivative of $\frac{e^x}{x^x}y$. Hence we have

$$\frac{e^x}{x^x}y=a\int\frac{\log x+1}{x^x}dx=-ax^{-x}+c.$$

The solution is therefore, $y=\frac{cx^x-a}{e^x}$.

Also solved by *G. B. M. ZERR*, and *L. C. WALKER*.

153. Proposed by J. SCHEFFER. A. M., Hagerstown. Md.

Find the equation of the loxodromic curve on an oblate spheroid.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia. Pa.

Take the figure to Problem 95, Calculus, page 79, No. 3, Vol. VII. Let $AG=\theta$, $EP=x$, $CO=b$, $OB=a$, and CG=an elliptical arc; $\angle QPN=\beta$. Then

$$PN=xd\theta,\ QN=ds=\sqrt{\frac{a^2-e^2x^2}{a^2-x^2}}dx,\ \frac{PN}{QN}=\tan\beta.$$

$$\therefore d\theta=\frac{\tan\beta\sqrt{(a^2-e^2x^2)}dx}{x\sqrt{(a^2-x^2)}}.\quad \text{Let } x=\frac{a\cos\varphi}{\sqrt{(1-e^2\sin^2\varphi)}}.$$

$$\therefore d\theta=\frac{(1-e^2)\tan\beta d\varphi}{\cos\varphi\,(1-e^2\sin^2\varphi)}.\quad \text{Let } \sin\varphi=y.$$